# Chiral behavior of baryon magnetic moments

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**Abstract.** The utility of chiral effective field theory, constructed in a manner in which loop contributions are suppressed as one moves outside the power-counting regime, is explored for baryon magnetic moments. Opportunities for the study of significant chiral curvature in valence and full QCD and the nontrivial behavior of strange- and light-quark contributions to the magnetic moment of the  $\Lambda$  baryon are highlighted.

**Keywords:** magnetic moment, lattice QCD, effective field theory, nucleon properties, hyperons **PACS:** 12.39.Fe, 12.38.Gc, 13.40.Em, 14.20.Dh, 14.20.Jn

### INTRODUCTION

It is a pleasure to have this opportunity to review some of the most interesting aspects of baryon structure on the occasion of Tony Thomas' 60th birthday. In reflecting on our collaborative work with Tony [1–70] it is clear that the focus must be on the development of finite-range regularised (FRR) chiral effective field theory ( $\chi$ EFT); a breakthrough founded on deep insights gained through an extensive foundation of modeling hadron phenomenology now vindicated through an analysis of the most recent lattice QCD results [71].

In this presentation, we draw on the lattice simulation results of the CSSM Lattice Collaboration for baryon electromagnetic form factors published in Ref. [72]. These results approaching the quenched chiral limit were made possible through the improved chiral properties of the FLIC fermion action [25, 73, 74, 75].

## THE PÁDE

Through the application of the Cloudy Bag Model to the extrapolation of lattice QCD results [1], it became abundantly clear that nearly all of the lattice QCD results of the time were outside of the power counting regime of chiral perturbation theory [60]. This lead to the development of chiral extrapolation techniques that contain the correct non-analytic behavior of chiral perturbation theory but take on the smooth almost linear behavior of baryon properties at moderately large quark masses. The first approach exploited a Páde [2]

$$\mu_{p(n)} = \frac{\mu_0}{1 - \chi_{p(n)} m_{\pi} / \mu_0 + \beta m_{\pi}^2},\tag{1}$$

which builds in the Dirac moment at moderately large  $m_{\pi}^2$  and has the correct leading non-analytic (LNA) behavior of chiral perturbation theory at small  $m_{\pi}$ ,  $\mu = \mu_0 + \mu_0$ 

 $\chi_{p(n)}m_{\pi}+\cdots$ , with  $\chi_{p(n)}$  a known model independent constant. While this approach was phenomenologically successful, it became clear that incorporating higher order terms and accommodating baryons having the coefficient  $\chi > 0$  was difficult [9, 10].

### FINITE RANGE REGULARISED EFFECTIVE FIELD THEORY

The solution was found in an a alternative regularisation of chiral effective field theory now known as Finite Range Regularisation where a regulator function is introduced to suppress the large momenta of effective degrees of freedom in chiral loop integrals [3, 6, 15, 32]. Mathematically equivalent to chiral perturbation theory to any finite order calculated, this series expansion has the additional feature of the suppression of loop-integral contributions at moderate pion masses in accord with lattice simulation results. Thus, this resummation of the chiral expansion will not make catastrophic errors outside of the power-counting regime.

Turning our focus now to the specific phenomenology of baryon magnetic moments, we employ the approach of Ref. [5] which includes the non-analytic behavior from photon couplings to  $\pi$  and K mesons in the presence of intermediate octet or decuplet baryons. Unphysical  $\eta'$  contributions are also included in the quenched analysis. Whereas Ref. [5] focused on the quark-sector contributions to baryon magnetic moments and their environment sensitivity, here we combine the sectors to study the chiral behavior of the proton and neutron magnetic moments.

As the lattice simulation results [72] are obtained in the quenched approximation, we exploit our discovery of a link between quenched QCD and full QCD through a replacement of the quenched meson cloud with the meson cloud of full QCD [4] via FRR  $\chi$ EFT [3]. The coefficients for unquenching baryon magnetic moments are taken from Ref. [76].

Figures 1 and 2 illustrate results for the proton and neutron respectively. A fit of FRR quenched  $\chi$ EFT (solid curve) to the FLIC fermion lattice points is illustrated. Here only the discrete momenta allowed in the finite volume of the lattice are summed in performing the loop integral. The long-dashed curve that also runs through the lattice results corresponds to replacing the discrete momentum sum by the infinite-volume, continuous momentum integral. Incorporating baryon mass splittings into the kaon loop contributions is essential – e.g., the contribution of  $\Sigma \to NK$  is almost doubled when the  $\Sigma - N$  mass splitting is included.

The removal of quenched  $\eta'$  contributions and the appropriate adjustment of  $\pi$  and K loop coefficients [76] provides the dot-dash curve of Fig. 1. This is our best estimate of the proton magnetic moment of full QCD where only the valence quarks couple to the electromagnetic current; i.e. a connected insertion of the electromagnetic current. This valence contribution can be thought of as a form of partially-quenched QCD where the sea quarks have the correct mass, but have neutral electric charges.

Finally, contributions from the disconnected insertion of the current are estimated via meson loop contributions to provide the proton magnetic moment in full QCD (fine dash-dot curve in Fig. 1). Figure 2 displays similar results for the neutron.

The calculation of valence properties in full QCD is a relatively easy task as the disconnected current insertions are not required. Moreover the chiral non-analytic behav-

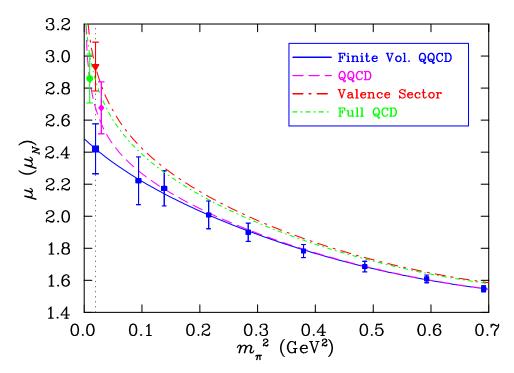


FIGURE 1. The magnetic moment of the proton. Curves and symbols are described in the text.

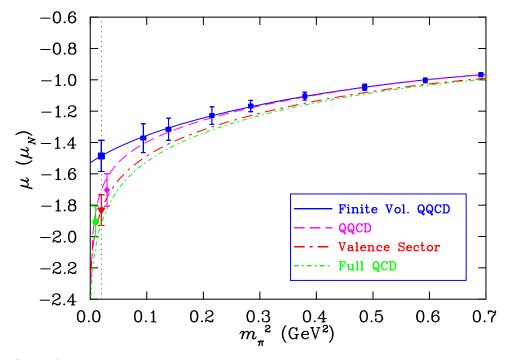
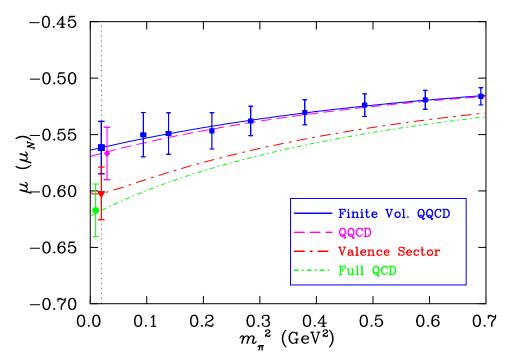


FIGURE 2. The magnetic moment of the neutron. Curves and symbols are described in the text.



**FIGURE 3.** The magnetic moment of the  $\Lambda$ . Curves and symbols are described in the text.

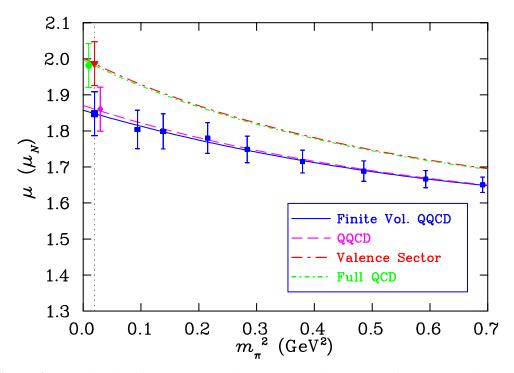
ior in valence QCD is strong as the screening effects from anti-quark couplings in the mesons are absent. Noting that disconnected current insertions vanish in isovector contributions, valence QCD and full QCD are equivalent for isovector quantities, making these observables ideal for revealing and understanding the meson cloud of baryons. We predict significant enhancements in the magnitude of proton and neutron magnetic moments on large-volume lattices as the chiral limit is approached.

The magnetic moment of the  $\Lambda$  baryon, illustrated in Fig. 3, is particularly interesting to study. In simple quark models the moment is provided solely by the strange constituent quark. However, these recent precise lattice results [72] reveal it to have a more interesting composition. First, we note how the magnetic moment has a clear dependence on the quark mass, represented by  $m_{\pi}^2$ . Whereas  $\pi\Sigma$  contributions survive in valence QCD, they vanish in quenched and full QCD, leaving kaon contributions to drive the non-analytic behavior. While there is little curvature from the kaons, we note that the unquenching of the kaon contribution from quenched to full QCD is significant.

Figure 4 illustrating the strange-quark contribution to the  $\Lambda$  magnetic moment reveals the  $m_\pi^2$  dependence to be predominantly due to an environmental dependence of the strange quark contribution. While the mass of the strange quark is held fixed in the simulations, the changing light quark mass leads to a change in the strange quark contribution. Such environmental effects are natural in terms of KN and  $K\Xi$  dressings of the Lambda and unquenching effects are predicted to be significant.

Figure 5 reveals the nontrivial role of light-quark contributions to the Lambda magnetic moment. Again,  $\pi\Sigma$  contributions are admitted in valence QCD [76], whereas kaon dressings drive the non-analytic behavior of quenched and full QCD.

In summary, FRR  $\chi$ EFT has provided many precise predictions for the electromagnetic properties of baryons in both valence and full QCD. With large-volume lattice



**FIGURE 4.** Contribution of the strange quark to the magnetic moment of the  $\Lambda$  normalized to unit charge.

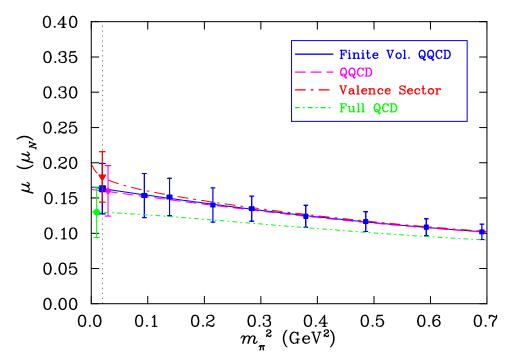
QCD simulations of valence QCD now at the fore of the field, it will be interesting to learn the success of these predictions and the extent to which our current understanding of the essential physics behind these observables is accurate.

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**FIGURE 5.** Contribution of a u or d quark to the magnetic moment of the  $\Lambda$  normalized to unit charge.

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